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A simulation model of temperature transitory
on rocks having different thermal inertia.

Analysis of the theoretical capacity of rock
discrimination by remote sensing data.

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1. Foreword and purpose of research.

Determination in the field of the nature of ground surface material is frequently time and effort consuming, and indeed is not always technically possible. Thus the possibility of effecting such analysis by remote sensing (by satellite or aeroplane) seem of great interest.

At least theoretically, it may be possible to deduce the nature of the material forming a terrain from its thermal characteristics, by measuring, by remote sensing, for example, its surface temperature at two different and significant moments of the day, when temperature value is close to minimum and to maximum. In fact it can be shown $[1]$, $[2]$, $[3]$, $[4]$ that the value of total daily temperature excursion of a soil (maximum value minus minimum value) is connected to the product $\varsigma \cdot c \cdot \lambda$ which is characteristic of the material being examined, where ς is density, c is specific heat, λ thermal conductivity.

On the whole this connexion is effectively recognizable only if the surface is composed of integral rocky material (i.e., has no surface stratum of degraded, fragmented or transported material), with a negligible humidity content, and not covered by any kind of vegetation $[5]$, $[6]$.

As a hypothesis a rocky surface of the above kind was envisaged, consisting in blocks of three different materials (see par. 4, below) and an analysis was made of the possibility of identifying the dividing lines between such blocks and, where possible, to form a first approximate idea of the kind of material forming the surface of the terrain.

This attempt at identification was carried out by simulation of the thermal dynamics of the soil surface during a day in which atmospheric conditions were average for the latitude in question (W. Europe).

As will be described in detail in par, 4, simulation was used to compute the values presented by the rocky surfaces consisting of blocks of three different materials, reference being made to spring or autumn months, at a latitude of about 40°-50°.

The line of separation between zones of different materials is delineated
by the range of temperature variation (difference between maximum and
minimum temperature daily values).

2. Numerical computation of temperature transitories in a solid of finite dimensions, composed of various materials, using the Crank-Nicholson finite--differences methods.

2.1 General.

Widely-known analytical methods permit the solution of the differential
equations governing diffusion of heat in solids [1], [7], [8], [9], [10], [11].
These methods have certainly a theoretical interest since they make it
possible to describe the physical phenomena with a general and organic approach
nevertheless in practice they can only be used for the solution of rather
simple problems, or as a quick way of obtaining rough preliminary results,
since the weightiness of the calculations rapidly becomes prohibitibe as
soon as the boundary conditions, or the shape of the solid becomes complicated.
The advent of computers produce a reduction in the importance of analytical
methods for the solution of the differential equations, which have gradually
been replaced by numerical methods, which are much more easily implemented by
the computer. A theoretical case of computation of temperature transitories
under periodic regime was resolved using a finite-differences mathematical
model (with time discretization according to the Crank-Nicholson method
and solution of the system for computation of the unknown quantities by
the Gauss-Seidel or surrelaxation method). For an exhaustive analysis of
such finite-differences methods, see [9], [10], [11].

2.2 Numerical Case Study.

A study was made of the temperature variations that would take place over
a period of time on an island, rectangular in shape, and consisting of
three different materials (fig. 1).

FIG. 1 - Plant of Island

It was assumed, as shown in fig. 1, that the separation surfaces between different materials were vertical planes having y=constant.

The island is supposed to be surrounded by the sea, the temperature of which, by a first approximation, is taken to be at a constant value of 295°K throughout the day. In order to operate with the finite-differences method, the island was divided, on the xy plane, into the network shown in Fig. 1.1.

FIG. 1.1 - Dimension and subdivision in blocks of Island

Along the z (depth) axis, temperature values are computed only for the first six metres, it being assumed that at such depths there is practically no diurnal temperature variation (it may be calculated with the numerical finite-differences methods, or even taking an approximate value from the analytical method, [1], that the daily temperature variation at such depth is of the order of 10^{-9}°K for rocky materials. The temperature at 6 m. depth is therefore assumed to be constant.

On the vertical plane the solid is therefore divided into 12 slices, each 0.5 m thick (fig. 1.2).

FIG. 1.2 - Subdivision of parallelepiped (island) into horizontal plane blocks .

In fig. 1.1 is can be seen that the general (not borderline) mesh has the dimensions (shown in Fig. I.2.I.), 10 m. and 20 m.,

, while borderline meshes have a perpendicular dimension at the every border of 40 cm. (A borderline mesh is one belonging to the perimeter of the Island).

This differentiation was introduced to make it possible to consider borderline meshes as being, at any given instant, in thermal equilibrium with the temperature of the sea ("waterline zone"). For such a border-line zone the order of magnitude of the perpendicular to border dimension was taken to be such that it would be permanently wetted by waves breaking on the coast.

Boundary and initial conditions.

The following boundary and initial conditions were assigned:
Boundary conditions:

a) Temperature constant, in time, at depth of 6 m

The temperature values at 6 m depth (values maintained constant in time) are computed taking heat flow in a vertical direction to be constant and of value equal to that of the mean geothermal flow of the Earth, i.e., approximately 1,5 $\frac{\mu \; cal}{cm^2 \; s}$ [12].

It is further assumed that the initial surface temperature is uniform all over the island, the value being 296°K.
(This temperature value was chosen because it was observed, by the simulation of the transitory, to be the mean daily value reached by the soil surface temperature, with the values which were assigned in the example to the parameters belonging to the heat equation, when the initial temperature transitory in the simulation is fulfilled and the periodic stationary regime is reached).
Temperature at depth 600 cm. can easily be found from equation:

$$\vartheta_{600} = \vartheta_{sup} + \frac{\phi}{\lambda_i} \cdot 600 \; cm \qquad (1)$$

where
λ_i = heat conductivity of the i-th material (cal/cm.s.°K)
ϑ_{sup} = surface temperature of material
ϑ_{600} = temperature at 600 cm depth
ϕ = geothermal flow (cal/cm².s)

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Geothermal flow was taken to be constant all over the island both because this was simpler, and in order to avoid the surface-temperature values computed during the transitory being influenced by the variability of the geothermal flow. In fact, in the case under consideration, the aim is to compute the influence of the various materials of which the island is composed, on the surface temperatures.

b) Heat flow imposed un surface of island.

With the exception of the narrow "waterline" strip running along the perimeter of the island and 40 cm wide, in which, as already stated, surface temperature is taken to be at the constant value of 293°K (which is a reasonable supposition since the thermal inertia of water is much greater than that of the materials forming the island), the net heat flow crossing unit ground surface in one second was supposed to be (fig. 2):

FIG. 2 – Heat exchanges at the surface

J (t) heat flow by radiation from sun (cal/cm^2.s) in the surface

As a first approximation for the sake of simplicity the heat flow is supposed to be sinusoidal, having a mean value of J . Hence the analytical expression of the heat flow reaching the surface by radiation from the sun will be of the following form:

$$J (t) = J (1 + \sin (\omega t))$$

where ω is pulsation of oscillation of solar radiation (1/s)

$$\omega = \frac{2\pi}{T} \qquad \text{where } T = 24 \text{ h, period}$$

$\varepsilon\sigma\vartheta\big|_{z=0}$ value of radiation of Earth's surface towards space as given by the Stefan-Boltzmann law (as a first approximation the temperature of the universe is taken to be $0°$ K).

$\lambda\dfrac{\partial\vartheta}{\partial z}\big|_{z=0}$ geothermal flow crossing ground surface each second.

c) Temperatures imposed in points belonging to lateral surface (excluding upper and lower surfaces) of the parallelipiped (island).

FIG. 3 - Section of island by plane x = const.(or y=const.)

Fig. 3 represents a section of the island with a plane $\underline{x}$ = constant, or $\underline{y}$ = constant.

Fig. 1.1 gives a plan of the island.

Let $\underline{S}\,(\underline{x},\,\underline{y},\,\underline{z})$ denote the set of points belonging to the parallelepiped being studied (Fig. 3).

The set of coordinates of the points forming the border of the island, 40 cm wide, shall have coordinates (x,y) belonging to a set $\overline{S}(x,y)$.

Now consider the points belonging to the lateral surface SL (x, y, z) of the parallelepiped and having Z < 6 m; i.e. SL (x,y,z) consists of the points with $(x,y) \in \overline{S}\,(x,y)$ and with $\underline{z}$ < 6 m.

Such points belong to the set denoted by SL in Fig. 3.1

FIG. 3.1 - Section of island by plane $x=$ const. (or y= const.)

Since there are reasons for believing that the areas belonging to SL which are closest to the surface (e.g. those with $o < z < 2m$) are affected by the influence of the sea temperature (mean temperature, it is supposed that the temperatures of such zones follow a pattern as a function of $\underline{z}$ of the type (fig. 4).

as regards

FIG. 4 - Initial temperature values of parallelepiped (Island)

In Fig.4

(1) initial temperature for zones belonging to SL.

(2') initial temperatures for zones composed of same material as above, belonging to S but not to SL.

It is supposed that at depths over 2m the influence of the temperature
of the sea is' not felt, this influence tending to lower mean temperatures
in zones immediately below the sea.

To sum up, it was supposed that zones belonging to SL presented two
temperature gradients in z; one for $0 < z \leqslant 2m$, and another for $z > 2m$ (fig. 4).
Lastly it was supposed (fixed potential boundary conditions), that the
temperature of zones belonging to the set SL remained constant in time.

Initial condition.

Initial surface and deep temperature values,
The initial value of surface temperature ϑ_o is fixed.

$$\vartheta_o(x, y, 0) = 296°K \quad \text{if } (x y) \notin \bar{S} \qquad \text{(non-bunder zone)}$$

$$\vartheta_o(x, y, 0) = 293°K \quad \text{if } (x, y) \in \bar{S} \qquad \text{(border zone)}$$

for points having $z > 0$, ϕ being geothermal flow and λ_i thermal
conductivity of the $\underline{i}$ - th material. We find

$$\vartheta_o(x, y, z) = \vartheta_o(x, y, 0) + \frac{\phi}{\lambda_i} z \qquad (3)$$

with $Z > 0$ _

if point P $(x, y, z) \notin$ SL, or if point P belongs to SL and has $z \geqslant 2m$.
On the contrary, if point P (x, y, z) has $0 < z < 2m$ and belongs to SL,
given that $z_2 = 2m$, we have the following initial temperature conditions

$$\vartheta_o(x, y, z) = \vartheta_o(x, y, 0) + \frac{\vartheta_o(x, y, z) - \vartheta_o(x, y, 0)}{z_2} \cdot z$$

$$(3.1)$$

Thus (3) and (3.1) given an analytical expression of the conditions
illustrated graphically in fig. 4.

3. Decription of the finite-differences mathematical model used to simulate the temperature transitory of the island.

On the (x-y) plane the island was subdivided into a network of (8x14)
zones (fig. I5.1).

In direction z analysis was carried out for the first 6 m in depth, dividing this depth into I2 zones (fig. 1.2).

In total the parallelepiped was divided into 8 x I4 x I2 = I344 elementary
parallelepipeds.

Each elementary parallelepiped not belonging to the border set SL thus has
the dimensions (x, y, z respectively) of 20 m x I0 m x 0.5 m (fig. 5).

FIG. 5 - Dimensions of elementary parallelepiped (not
belonging the border)

the island as a whole having the dimensions I20.8 m x I20.8 m x 6 m.
As regards the form of discretization adopted, the following observations
should be made:

1) initially, since it was not easy to estimate a priori the magnitude
of heat flows, simulation was carried out for a parallelepiped (island)
consisting of elementary parallelepipeds of the same dimensions as the
above but consisting, in directions x, y, z, respectively, of 12 x 20 x 20
elementary parallelipiped (a total of 4 800).

It was however found that temperature variations in direction x,y
being very "slow", and the temperature below 5.5 m depth not varying
significantly over time intervals of the order of one week, it was
possible to reduce the number of elements to 1 344, as described above,
without loss of precision.

2) The computation programme was prepared in such a way as to be adaptable,
by very small and rapid modifications, to run . with any number of
elementary parallelipipeds having any given dimensions. The model is
also suitable for use with a set of elementary parallelipipeds which at
the limit might be composed each of different materials.

In the model, as a preliminary approximation, it was felt possible to
neglect some elements which undoubtedly affect surface temperature, the
most important being [2],[4] : soil humidity (taken to be zero); evapora-
tion from soil; possible presence of degraded or fragmented material, or
of surface vegetation; possible wind near ground surface; atmospheric
temperature above the ground surface and air humidity; roughness of ground
surface.

Equations governing heat conduction in a solid (3-dimension model).

The most general differential equation governing heat conduction in a solid
(in the absence of phase transformations) is [1], [2]:

$$\rho c \frac{\partial \vartheta}{\partial t} = \frac{\partial}{\partial x}\left(\lambda \frac{\partial \vartheta}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda \frac{\partial \vartheta}{\partial y}\right) + \frac{\partial}{\partial z}\left(\lambda \frac{\partial \vartheta}{\partial z}\right) + q$$

(4)

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q being interior heat generation per unit volume and time

Having to apply (4) to compute temperatures of a solid of finite
dimensions, (4) can be approximated with the corresponding finite-
-differences equation [11].
The reasoning whereby such an equation is obtained is as follows:
considering the general case of a body fomed by elementary parallelipipeds
having dimensions x, y, z differing from each other .
As a generalization, further supposing that the dimensions of each
elementary parallelipiped differ from those of the others (fig. 6).

. FIG. 6 — Blocks formed by two elementary parallelepipeds,
each with different dimensions

With reference to the elements of fig. 6 compute the heat flow from
zone N to zone C.
In differential form the heat-flow equation is given by

$$q = - \lambda \frac{\partial \vartheta}{\partial x} \qquad\qquad (5)$$

taking the joining line NC as x axis (fig. 6).
The minus sign in (5) appears because heat flow versus is from the higher-
-temperature to the lower-temperature zone.
Defining as

$$R = \frac{1}{\lambda}$$

R thermal resistivity (inverse of thermal conductivity).
The equivalent thermal resistivity R_{NC} between point N and point C
is (in analogy to the sum of two resistences in series in an electrical
circuit).

$$R_{NC} = \frac{R_N \, \Delta y_N + R_c \, \Delta y_c}{\Delta y_N + \Delta y_c} \tag{7}$$

where R_N = resistivity of zone N, R_C that of zone C

$\Delta y_{N}, \, \Delta y_c$ dimensions of parallelipiped in direction $\underline{y}$

The equivalente thermal conductivity between zone N and zone C is (7):

$$\lambda_{NC} = \frac{1}{R_{NC}} = \frac{\Delta y_N + \Delta y_c}{R_N \Delta y_N + R_c \Delta y_c} = \frac{\Delta y_N + \Delta y_c}{\frac{\Delta y_N}{\lambda_N} + \frac{\Delta y_c}{\lambda_c}} = \frac{\lambda_N \lambda_c \, \Delta y_N \, \Delta y_c}{\lambda_N \Delta y_c + \lambda_c \Delta y_N} \tag{7.1}$$

Note that, since we are dealing with resistivity, (7) and (7.1) are
obtained for two parallelipipeds of unit base area, hence for simplicity
with

FIG. 7 - Blocks formed by parallelepipeds of unit base area

However, for the general case of fig. 7, we find that the total resistence
between point N and point C has the value

$$\overline{R}_{TOT} = R_{NC} \, \frac{\Delta y_N + \Delta y_c}{2} \cdot \frac{1}{\Delta x_c + \Delta z_c} = \frac{\lambda_N \Delta y_c + \lambda_c \Delta y_N}{2 \lambda_N \lambda_c} \cdot \frac{1}{\Delta x_c \, \Delta z_c} \tag{5.1}$$

the total heat conductivity between point N and point C is, from (5.1)

$$G_{TOT} = \frac{1}{\overline{R}_{TOT}} = \frac{2 \lambda_N \, \lambda_c}{\lambda_N \Delta y_c + \lambda_c \Delta y_N} \cdot \Delta x_c \cdot \Delta z_c \tag{5.2}$$

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FIG. 8 – General blocks (c) with surrounding blocks
(general dimensions)

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The heat passing between zone N and zone C per unit time is, acccording to (5) and (5.2) (electrical analogues):

$$Q_{N\overline{C}} = \overline{G}_{T \cdot r}' \left(\vartheta_N - \vartheta_C\right) = \frac{2\lambda_N \lambda_c}{\lambda_N \Delta y_c + \lambda_c \Delta y_N} \cdot \Delta x_c \, \Delta z_c \cdot \left(\vartheta_N - \vartheta_C\right) \qquad (5.3)$$

where Δx_c and Δz_c are the dimensions, (with respect to $\underline{x}$ and $\underline{z}$), of the face perpendicular to y axis.

In general, if we consider an elementary parallelipiped, C, surrounded by zones (parallelipipeds) of dimensions in general differing from each other, the total heat flow q_c, entering in C, each second from the surrounding zones is, according to $\left(5.2\right)$ – $\left(\text{Fig 8}\right)$)

$$q_c = 2\left[\frac{\lambda_N \lambda_c \, \Delta x_c \Delta z_N}{\lambda_N \Delta y_c + \lambda_c \Delta y_N} \cdot \left(\vartheta_N - \vartheta_c\right) + \frac{\lambda_s \lambda_c \, \Delta x_c \Delta z_s}{\lambda_s \Delta y_c + \lambda_c \Delta y_s} \cdot \left(\vartheta_s - \vartheta_c\right) + \frac{\lambda_c \lambda_E \, \Delta y_c \Delta z_c}{\lambda_E \Delta x_c + \lambda_c \Delta x_E} \cdot \right.$$

$$\cdot \left(\vartheta_E - \vartheta_c\right) + \frac{\lambda_c \lambda_w \, \Delta y_c \Delta z_c}{\lambda_w \Delta x_c + \lambda_c \Delta x_w} \left(\vartheta_w - \vartheta_c\right) + \frac{\lambda_c \lambda_{AS} \, \Delta y_c \Delta x_c}{\lambda_{AS} \Delta z_c + \lambda_c \Delta z_{AS}} \left(\vartheta_{AS} - \vartheta_c\right) +$$

$$\left. + \frac{\lambda_c \lambda_{AI} \, \Delta y_c \Delta x_c}{\lambda_{AI} \Delta z_c + \lambda_c \Delta z_{AI}} \left(\vartheta_{AI} - \vartheta_c\right) \right] \qquad . \qquad (8)$$

The heat accumulating in one second inside the elementary parallelipiped C causes, in time $\underline{t}$, a variation of the temperature of C which is given by (4)

$$\Delta \vartheta_c = \frac{\partial \vartheta_c}{\partial t} \cdot \Delta t = \frac{1}{\rho c} \cdot q_c \cdot \Delta t \qquad (9)$$

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To the finite differences $\dfrac{\partial \vartheta}{\partial t}$ is expressed as

$$\frac{\vartheta(x_c, y_c, z_c, t+\Delta t) + \vartheta(x_c, y_c, z_c, t)}{\Delta t}$$

For (8) and (9) we may thus write, taking coordinate j to express the discrete value of x, i that of y, and k that of z:

$$\frac{\vartheta_{t+\Delta t} - \vartheta_t}{\Delta t} = \frac{1}{\varsigma c}\left(\frac{1}{\Delta y}\left(2\frac{\lambda\,\lambda_{i-1}}{\lambda\,\Delta y_{i-1} + \lambda_{i-1}\Delta y}\cdot\frac{\vartheta_{i-1,t+\Delta t} + \vartheta_{i-1,t}}{2} + 2\cdot\frac{\lambda\,\lambda_{i+1}}{\lambda\,\Delta y_{i+1} + \lambda_{i+1}\Delta y}\right.$$

$$\left.\cdot\frac{\vartheta_{i+1,t+\Delta t} - \vartheta_{i+1,t}}{2}\right) + \frac{1}{\Delta x}\left(2\cdot\frac{\lambda\,\lambda_{j-1}}{\lambda\Delta x_{j-1} + \lambda_{j-1}\Delta x}\cdot\frac{\vartheta_{j-1,t+\Delta t} + \vartheta_{j-1,t}}{2} + 2\frac{\lambda\lambda_{j+1}}{\lambda\Delta x_{j+1} + \lambda_{j+1}\Delta x}\right.$$

$$\left.\cdot\frac{\vartheta_{j+1,t+\Delta t} - \vartheta_{j+1,t}}{2}\right) + \frac{1}{\Delta z}\left(2\frac{\lambda\lambda_{k-1}}{\lambda\Delta z_{k-1} + \lambda_{k-1}\Delta z}\cdot\frac{\vartheta_{k-1,t+\Delta t} + \vartheta_{k-1,t}}{2} + 2\frac{\lambda\,\lambda_{k+1}}{\lambda\Delta z_{k+1} + \lambda_{k+1}\Delta z}\cdot\frac{\vartheta_{k+1,t}}{2}\right.$$

$$\tag{10}$$

In (IO) in order to simplify notation the index in the expressions of Δx, Δy, Δz, λ, ς, c, ϑ has been taken as understood when equal to i, j, k .

Equation (IO) makes it possible to compute the temperatures of the single elementary parallelipipeds once the boundary conditions and the initial conditions of the system are known. The unknown temperatures $\vartheta(i, j, k, t+\Delta t)$ are computed from the known temperatures $\vartheta(i; j, k, t)$ at time t , using a system of equations of type (IO). This system is resolved by the iterative surrelaxation method [2] , [11] .

The boundary heat-flow condition imposed at the surface of the solid, which is expressed in the differential form (2) , is expressed in the finite-differences form as follows.

Let us consider a generic parallelipiped on the surface of the solid (z=0), but not on the borderline, with geometrical center $\bar{C} = \bar{C}\left(\bar{x}, \bar{y}, \frac{\Delta z}{2}\right)$ (fig. 9)

FIG. 9 – Heat exchanges in general surface blocks

the heat equation for parallelipipeds belonging to the surface is expressed as:

$$\Delta x \, \Delta y \, \Delta z \left(\beta c \, \frac{\Delta \vartheta}{\Delta t}\bigg|_{\tau = t + \frac{\Delta t}{2}}\right) = \frac{J(t+\Delta t)+J(t)}{2}\Delta x \Delta y - \varepsilon \sigma \left(\frac{\vartheta(t+\Delta t)+\vartheta(t)}{2}\right)^4 \Delta x \cdot \Delta y$$

$$+ \lambda_{eq_z}\frac{\partial \vartheta}{\partial z}\bigg|_{\bar{T}=t+\frac{\Delta t}{2}} \Delta x \, \Delta y + \lambda_{eq_x}\frac{\partial \vartheta}{\partial x}\bigg|_{\tau = t+\frac{\Delta t}{2}} \Delta y \, \Delta z + \lambda_{eq_y}\frac{\partial \vartheta}{\partial y}\bigg|_{\tau = t+\frac{\Delta t}{2}} \Delta x \, \Delta z$$

τ time variable

β, c, ϑ referred to point $\bar{C}$, geometric center.

(11)

In discrete form (11) is expressed by the following finite-differences equation, which is general, valid even in the case of z and λ varying with depth:

$$\frac{\vartheta_{t+\Delta t}-\vartheta_t}{\Delta t} = \frac{1}{\beta c}\left[\frac{J_{t+\Delta t}+J_t}{2}\cdot\frac{1}{\Delta z} + 2\frac{\lambda\lambda_z}{\lambda\Delta z_z + \lambda_z \Delta z}\cdot\frac{1}{\Delta z}\cdot\left(\frac{\vartheta_{z,t+\Delta t}+\vartheta_{z,t}}{2} - \right.\right.$$

$$\left. - \frac{\vartheta_{t+\Delta t}+\vartheta_t}{2}\right) + \frac{1}{\Delta x}\cdot\left(\frac{2\lambda\lambda_{i-1}}{\lambda\Delta x_{i-1}+\lambda_{i-1}\Delta x}\cdot\left(\frac{\vartheta_{i-1,t+\Delta t}+\vartheta_{i-1,t}}{2} - \frac{\vartheta_{t+\Delta t}+\vartheta_t}{2}\right) - \right.$$

$$\left. - \frac{2\lambda\lambda_{i+1}}{\lambda\Delta x_{i+1}+\lambda_{i+1}\Delta x}\cdot\left(\frac{\vartheta_{i+1,t+\Delta t}+\vartheta_{i+1,t}}{2} - \frac{\vartheta_{t+\Delta t}+\vartheta_t}{2}\right)\right) + \frac{1}{\Delta y}\left(\frac{2\lambda\lambda_{j-1}}{\lambda\Delta y_{j-1}+\lambda_{j-1}\Delta y}\cdot\right.$$

$$\left(\frac{\vartheta_{j-1,t+\Delta t}+\vartheta_{j-1,t}}{2} - \frac{\vartheta_{t+\Delta t}+\vartheta_t}{2}\right) - \frac{2\lambda\lambda_{j+1}}{\lambda\Delta y_{j+1}+\lambda_{j+1}\Delta y}\cdot\left(\frac{\vartheta_{j+1,t+\Delta t}+\vartheta_{j+1,t}}{2} - \frac{\vartheta_{t+\Delta t}+\vartheta_t}{2}\right)\right)$$

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$$- \varepsilon \sigma \left(\frac{\vartheta_{t+\Delta t} + \vartheta_t}{2} \right)^4 \cdot \frac{1}{\Delta z} \Bigg]$$

;

$$(12)$$

In (I2) to simplify notation, the index is taken to be understood in the expression of $\Delta x, \Delta y, \Delta z, \lambda, \rho, c, \vartheta$ when equal (respectively) to i, j, l.

It should be observed that the finite-differences method thus makes it possible to attain a reasonably simple solution of the heat equation also for a non-linear problem such as that studied here, furthermore in a region formed of elements of different materials and dimensions.

The solution of such a problem by classical analytical methods would present monstrous difficulties of computing. Indeed, it may be stated that, since it is a non-linear problem, it cannot in practice be resolved by analytical traditional methods.

4. Description of simulation of the thermal transitory and analysis of results obtained.

Figs. 1.1 and 1.2 show the region in study which consists of three differing materials. The width in the $(\underline{x},\underline{y})$ plane of each of the three zones is respectively 40.8 m; 40 m, 40.8 m.

The first zone consists of dolomite, the second of granite, the third of dry clay (i.e. with practically-negligible water content).

It was supposed that the net solar heat flow $\underline{J}$ had a mean value of I5 m. .cal/cm^2 s, which is a reasonable value for locations around Latitude 40° [3] .

The following values were assigned to the parameters characteristic of the heat equations. They are roughly the mean for the materials in question [3], [I3] .

Dolomite

heat conductivity $\lambda = $ IO^{-2} cal/cm.s. °K

density ρ = 2.6 g/cm^3

specific heat c = 0.23 cal/gK

Granite

conductivity λ = 7.10^{-3} cal/cm.s.$^\circ$K.

density ρ = 2.52 g/cm^3

specific heat c= 0.20 cal/g.$^\circ$K

Clay

conductivity = 3.6.10^{-3} cal/cm.s.$^\circ$K.

density = J.8 g/cm^3

specific heat c= 0.20 cal/g.$^\circ$K

The emissivity of the rock surface was taken to be ε = 0.92   ($^{\circ\circ}$) for
all three materials; furthermore it was supposed that of the heat irradiated
toward the sky from the surface of the island, 25% would be retained by
the atmosphere, whilst 75% would pass beyond the barrier of the atmosphere
and be irradiated toward the universe.

The transitory was simulated starting from the initial condition described
in par. 2, simulation being protracted for 110 h. It was found that after
3 days (72h) the initial transitory could practically be taken to be
concluded, a permanent periodic regime having been reached.

From simulation it was deduced that heat exchanges in the x, y direction
are very small compared with heat exchanges taking place at the surface
due to heat flows in direction z.

In practice, for a given material, temperature differences between points
of the surface reach at most 7/100°K (in dolomite) (Tab. A, Tab. B)   at
any given instant.

Tab A   gives surface temperatures computed at a time approximately corre-
sponding to a minimum of the mean value of the surface temperatures (96h

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($^{\circ\circ}$) NOTE: the value of emissivity was taken to be equal for all three
materials considered, since these three types of rock effectively have rather
similar mean emissivity values [I3] , though these may vary   in
particular cases over a certain range.

It was thus felt that it would have been arbitrary to introduce any
differentiation based on emissivity values for the three rocks under
consideration.

after start of transitory). From top to bottom, zones are: dolomite, granite, clay.

TAB. A – Surface temperatures at time approximately
corresponding to a maximum of mean surface temperature

Tab. B presents temperatures of the zones at a time approximatively corresponding to a maximum of the mean value of surface temperature (IO7 h after start of transitory).
If we consider the central with respect to direction surface zones, i.e. close to the line VV of fig. IO.

TAB B – Surface temperature at time approximately corre-
sponding to a maximum of mean surface temperature

we find the following temperature values, from top to bottom (i.e. from N to S) excluding border zones (TABLE I).

TAB. 1 – temperature values along VV line of fig. IO

On the other hand, if we consider the horizontal sections traced
approximately at the centre with respect to the y axis of the single
homogeneous zones, in the E-W direction (lines OO, O'O', O"O" of fig.
IO.1) still excluding border zones, we obtain the temperature values
of TABLE II

FIG. IO – Vertical axis (VV) of the Island

TABLE II

TABLE II – temperature values along OO, O'O', O"O" lines of
fig. IO.1

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FIG. 10.1 - Horizontal axis (OO, O'O', O"O") of each homogeneous
zone of the Island

As we can be seen in table I and II the passage from one material to
another is marked by considerable variations of surface temperature values
T and of T, moving from North to South, while surface temperature and T
variations are minimal the same material both in the East-West and in the
North-South directions (see also figs. IO and 10.1).

Thus in the case studied, each zone may for all practical purposes be
considered to be nearly isolated from the surrounding zones.

FIG. 11 shows how, by thermal inertia measurements (i.e.,in practice by
measuring the difference ΔT between the daily maximum and minimum tempera-
tures), it is possible, in the simplified hypothetical situations considered
in this study, to identify materials of differing thermal inertias. It
must be noted that in this study materials having widely-differing thermal
inertias were chosen.

In fact, for dolomite the value is $P = 0.0773$ cal/cm^2 $\sqrt{s}$ 'K; for granite
0.0594 cal/cm^2 $\sqrt{s}$ 'K; for clay 0.0360 cal/cm^2 $\sqrt{s}$ 'K (thermal inertia
$P = \sqrt{\lambda \jmath c}$).

FIG. 11 _Daily temperature variations

5. CONCLUSION.

This report deals with the simulation of an ideal but significant case of temperature transitory on the surface of an island consisting of different rocks.

Assuming certain simplifying hypotheses (listed in par. 3), numerical simulation has shown the theoretical possibility of recognizing, in the example dealt with, the transition from blocks consisting of one material to blocks consisting of another material having different thermal inertia by remote sensing of surface-temperature values, given the sensivity of present-day temperature sensors.

Measurements must be made at least two different times of day; and if measurements are to be as significant as possible, one measurement should be made when the surface temperature values are close to minimum and the other when close to maximum.

It has in fact been shown that the difference between maximum and minimum value of the temperature of ground surface during the day is linked to the thermal inertia value of the material of which the rock is formed.

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TABLE OF FIGURES

290.0	290.0	290.0	290.0	290.0	290.0	290.0	290.0
290.0	294.50005	294.51250	294.51252	294.51252	29'.51250	294.50005	290.0
290.0	294.54904	294.56164	294.56164	294.56164	294.56164	294.54904	290.0
290.0	294.54916	294.56177	294.56179	294.56179	294.56177	294.54916	290.0
290.0	294.54982	294.56241	294.56243	294.56243	294.56241	294.54982	290.0
290.0	294.16209	294.17237	294.17239	294.17239	294.17237	294.16209	290.0
290.0	294.16280	294.17311	294.17312	294.17312	294.17311	294.16280	290.0
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290.0	290.0	290.0	290.0	290.0	290.0	290.0	290.0
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290.0	301.27470	301.28823	301.28823	301.28823	301.28823	301.27470	290.0
290.0	301.27483	301.28837	301.28838	301.28838	301.28837	301.27483	290.0
290.0	301.27554	301.28907	301.28908	301.28908	301.28907	301.27554	290.0
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290.0	304.13967	304.14681	304.14681	304.14681	304.14681	304.13967	290.0
290.0	290.0	290.0	290.0	290.0	290.0	290.0	290.

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TAB. I –

	T_{min} (°K)	T_{max} (°K)	$\Delta T = T_{max} - T_{min}$ (°K)
DOLOMITE	294.512	301.235	6.723
	294.552	301.288	6.726
	294.562	301.288	6.726
	294.562	301.289	6.727
GRANITE	294.172	302.171	7.999
	294.173	302.171	7.998
	294.173	302.171	7.998
	294.174	302.172	7.998
CLAY	292.965	304.174	11.209
	292.966	304.175	11.209
	292.966	304.175	11.209
	292.940	304.147	11.207
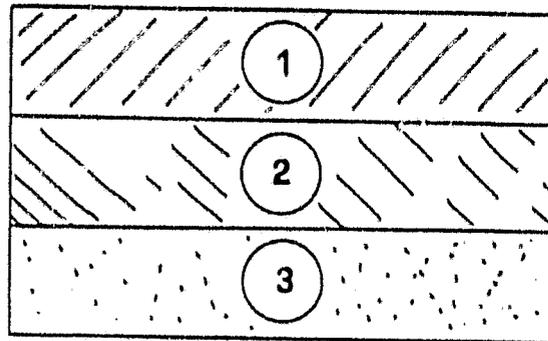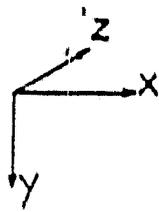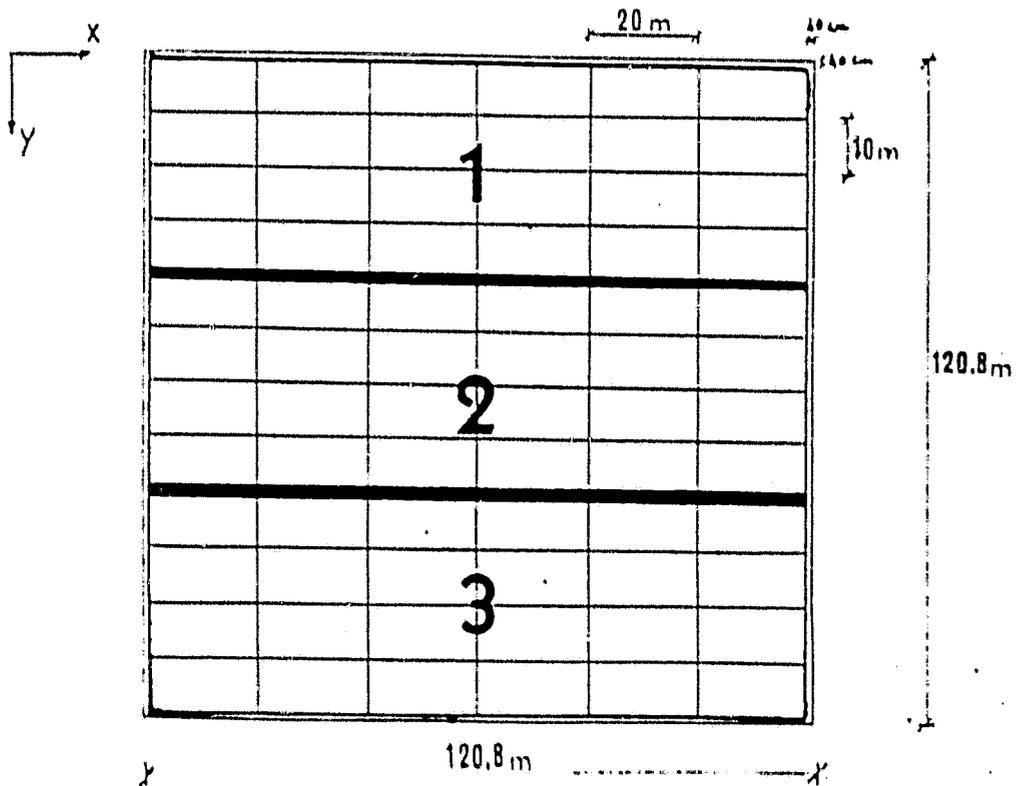
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FIG 1

FIG 1,1

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X
Y
Z

① ///////////

② ///////////

③

6 m

120.8 m

120.8 m

FIG 1.2

X
Y

20 m

10 m

FIG 1.2.1

$J(t)$

$\varepsilon \sigma \vartheta^4$

X
Y
Z

dz

$\lambda \cdot \dfrac{\partial \vartheta}{\partial z}$

1

1

1

FIG 2

29

FIG 3

FIG 3.1

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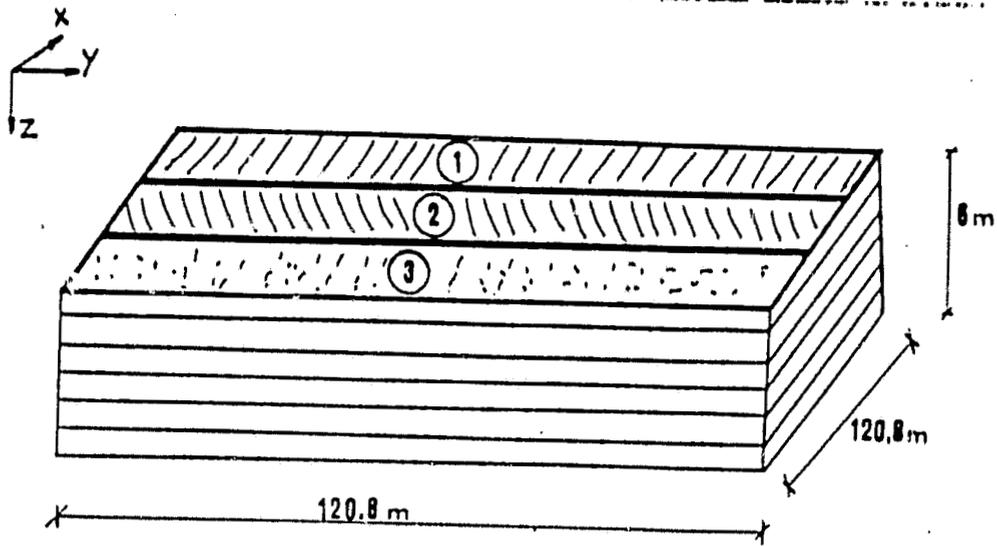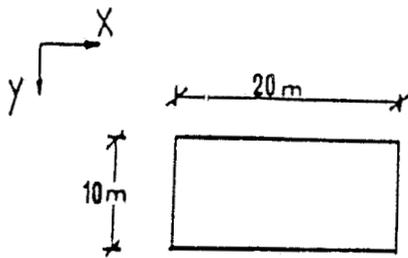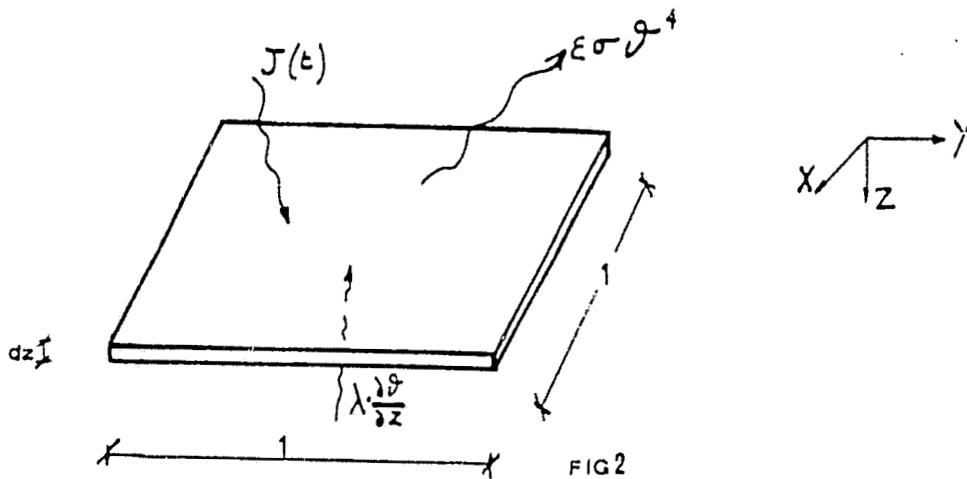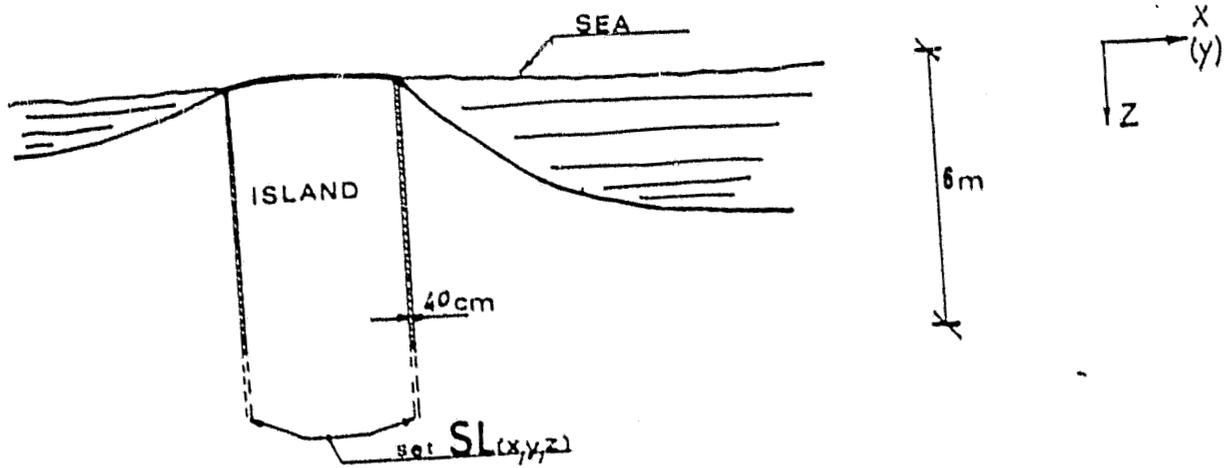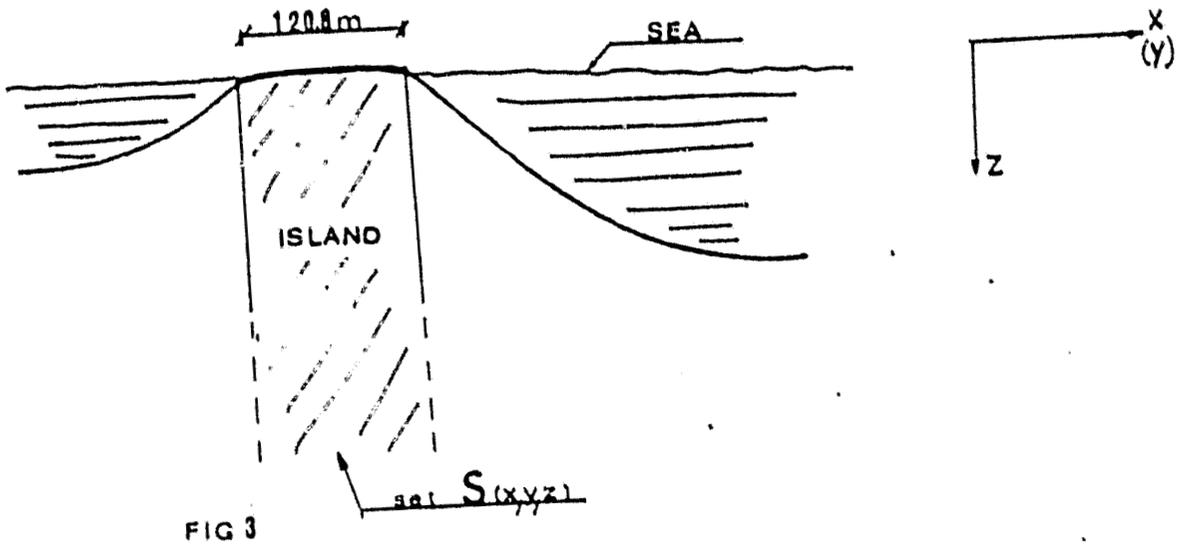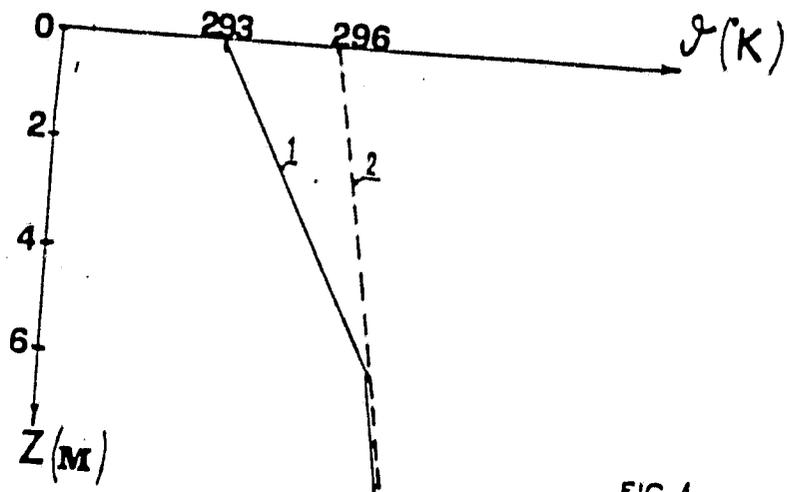
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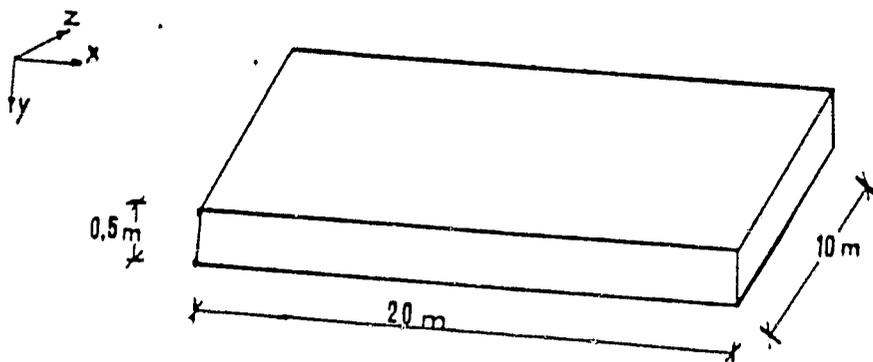
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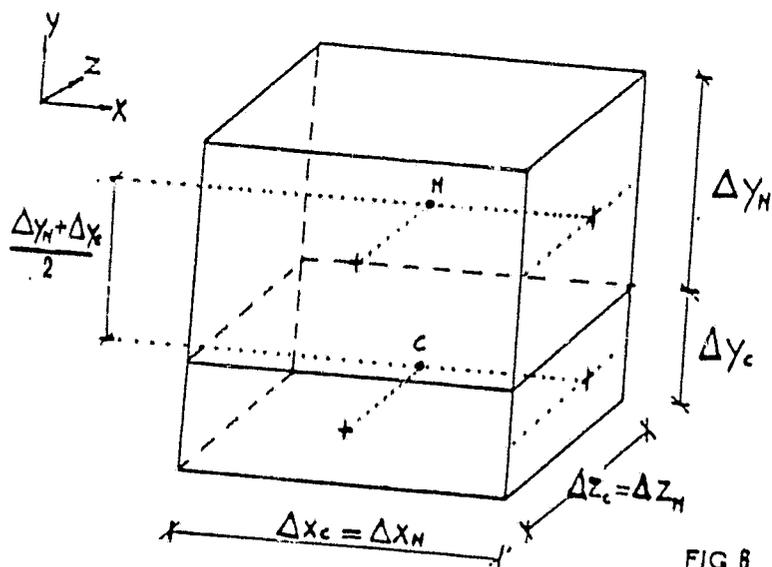
N UPPER BLOCK GEOMETRIC CENTER

C LOWER BLOCK GEOMETRIC CENTER

FIG 8

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FIG 7

$$\Delta y_H$$

$$\frac{\Delta y_H + \Delta y_c}{2}$$

$$\Delta y_c$$

FIG 8

FIG 9

FIG 10

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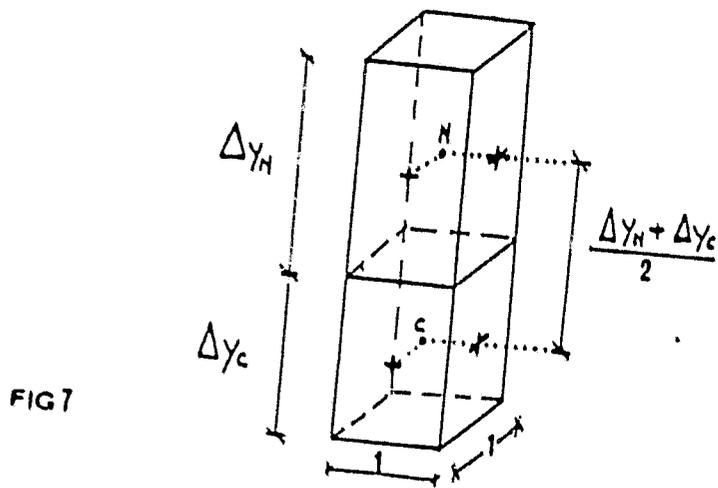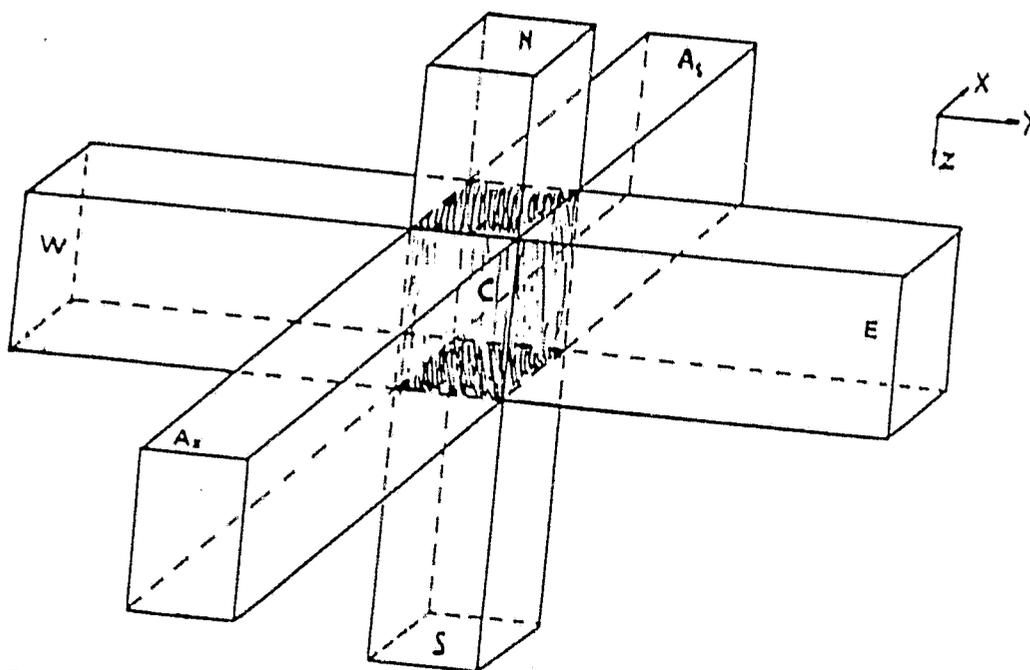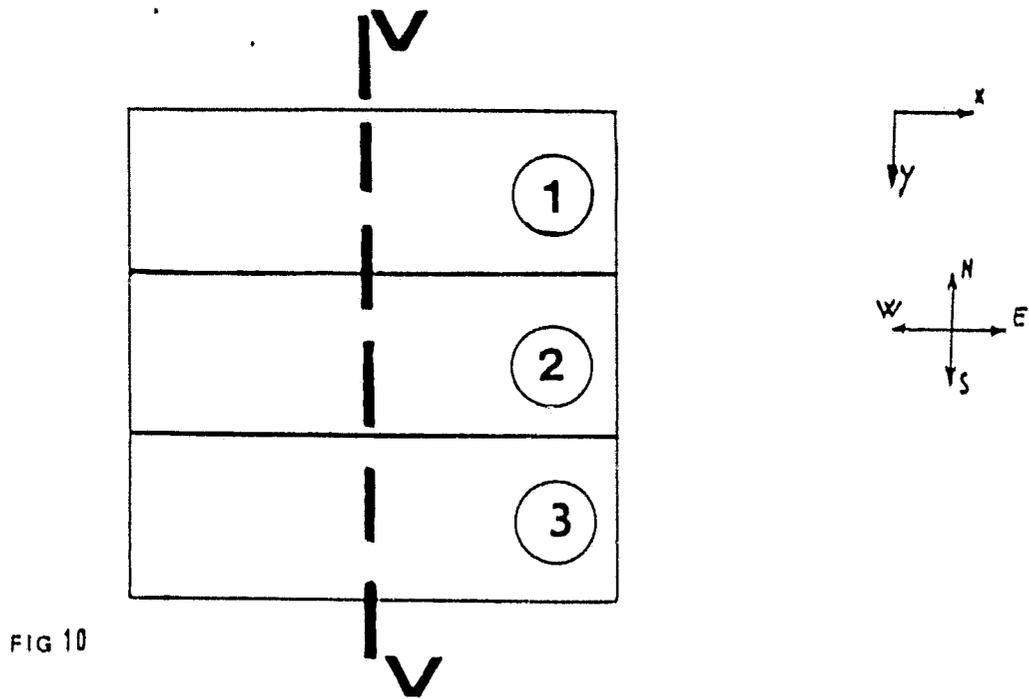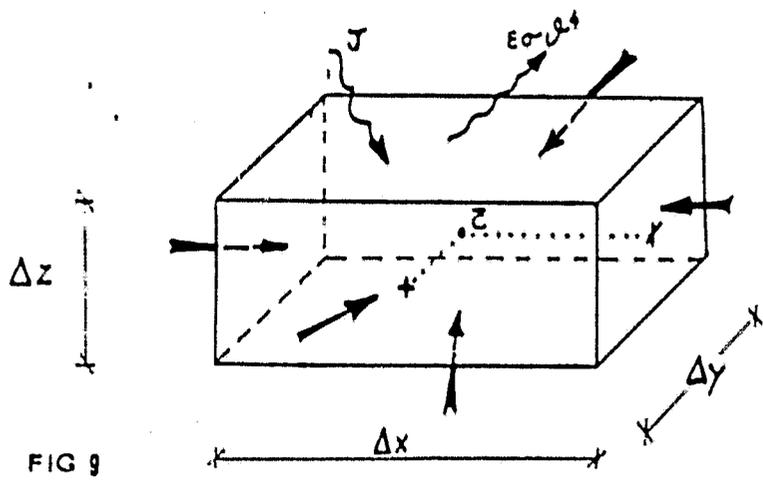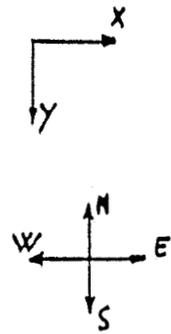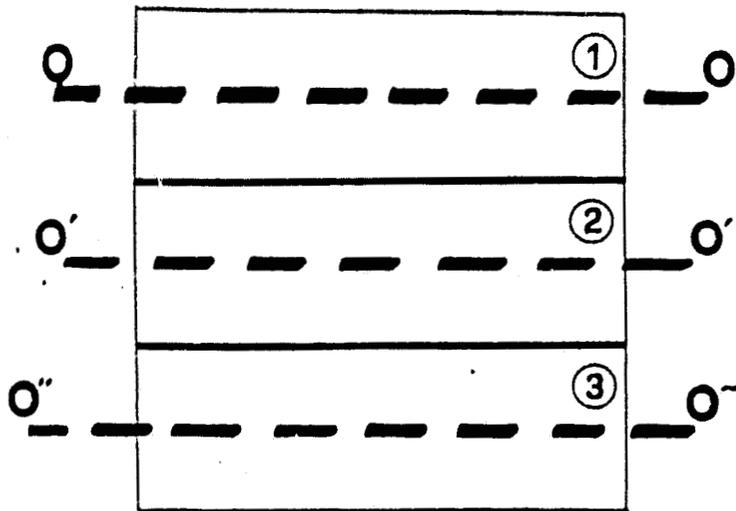
FIG 10.1

DAILY: TEMPERATURE VARIATIONS

dolomie
$\lambda = 0.01$ CAL/CM s $^\circ$K
$\rho = 2.6$ g/CM^3
$C = 0.23$ CAL/g $^\circ$K

granite
$\lambda = 0.007$
$\rho = 2.52$
$C = 0.2$

clay
$\lambda = 0.0036$
$\rho = 1.8$
$C = 0.2$

$\Delta T = T_{max} - T_{min}$ ($^\circ$K)

$[P] = [CAL/CM^2 \; K \sqrt{S}]$

P=.0036

P=.0594

P=.0773

ZONE

b.L dolomie   granite   clay   b.l.

b.l. ebard's line

FIG 11

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